FACULTY OF SCIENCE AND EDUCATION

Polynomial Curve Fitting Using Vandermonde Determinant

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Declaration

I **Diiri James Emmanuel,** hereby declare that this research is my own original work unless otherwise cited, and where such has been the case reference has been stated and that the same work has not been submitted for any award in any other university or other tertiary institute of higher education.

Signature…………………………………

Date…………..…………………

Diiri James Emmanuel.

Approval

This dissertation has been submitted for examination with the approval of my University Supervisor.

Signature………………………………….

Date……………………………

Dr. Asaph Keikara Muhumuza

Dedication

I dedicate this research to my dear parents Mr. Diiri James and Mrs. Muwanguzi Rebecca who through their tireless efforts I have become what I am today.

Acknowledgement

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Abstract

Matrices whose rows (or columns) consists of monomials of sequential powers are called Vandermonde matrices and can be used to describe several useful concepts and have properties that can be helpful for solving many kinds of problems. In this report we will discuss this matrix and some of its properties as well as a generalization of it and how it can be applied to polynomial curve fitting.

Demonstration of how to generate a polynomial curve fit is shown. The method used here is the least squares method where the Vandermonde determinant is used to fit an n-degree polynomial to a given data set.

Contents

Notations

The following notations will be used throughout the proposal unless defined otherwise

N, Z, R, C-The set of natural, integers, real and complex numbers

x, v-Bold, roman lower letters denote vectors

X, V, M-Bold, uppercase letters denote matrices

$C_{i,j}, M_{i,j}$-Element on the *i*-th row and *j*-th column of M

$M_{\cdot,j}$-Column vector of all elements from the *j*-th column of M

$C_{i,\cdot}$-Row vector of all elements from the *i*-throw of M

$\left[a_{ij}\right]^{nm}$-$ij - n \times m$matrix with element a_{ij} in the *i*-th row and *j*-th column

$V_{nm}(x)$- $n \times m$ Vandermonde matrix with respect to $x \in R^n$

$V(x) = V_{nn}(x)$- n-square Vandermonde matrix with respect to $x \in R^n$

$det\big(V(x)\big) = v_n(x)$- Determinant of n-square Vandermonde matrix

INTRODUCTION

1.1: Background

The Least Squares method was discovered by *Gauss* in *1795*. It has since become the principal tool to reduce the influence of errors when fitting models to given observations. Today, applications of least squares arise in a great number of scientific areas, such as statistics, geodetics, signal processing, and control (Åke Björck, 1996).

In the last 20 years there has been a great increase in the capacity for automatic data capturing and computing. Least squares problems of large size are now routinely solved. Tremendous progress has been made in numerical methods for least squares problems, in particular for generalized and modified least squares problems and direct and iterative methods for sparse problems. Until now there has not been a monograph that covers the full spectrum of relevant problems and methods in least squares (Åke Björck, 1996).

The method of least squares grew out of the fields of Astronomy and geodesy, as mathematicians and sccientists sought to provide solutions to the challenges of navigating the Earth's oceans during the age of exploration. The accurate desciption of the description of the behaviour of celestrial bobies was the key to enabling ships to sail in open seas, where sailors could nolonger rely on land sightings for navigation (Least_squares#History, 2021) (Wolberg, J., 2005) (Kariya & Kurata, 2004).

The method was the culmination of several advances that took place during the course of the eighteenth century:

The combination of different observations as being the best estimate of the true value; errors decrease with aggregation rather than increase, perhaps first expressed by Roger Cotes in 1722.

The combination of different observations taken under the same conditions contrary to simply trying one's best to observe and record a single observation accurately. The approach was known as the method of averages. This approach was notably used by Tobias Mayer while studying the librations of the moon in 1750, and by Pierre-Simon Laplace in his work in explaining the differences in motion of Jupiter and Saturn in 1788.

References

Åke Björck. (1996). *Numerical Methods for Least Squares Problems.* Linköping University, Linköping, Sweden: Society of Industtrial and Applied Mathematics.

Asaph Keikara Muhumuza. (2020). *Extreme points of the Vandermonde determinant in numerical approximation, random matrix theory and financial mathematics.* Sweden: Malardalen University.

Gergonne, J. D. (1815). The application of the method of least squares in the interpolation of sequences. *Historia Mathematica* , 439-447.

Hughes, T. (2020, August 9th). *The Vandermonde determinant, a Novel proof*. Retrieved MAy 19th, 2022, from Towards Data Science: https://towardsdatascience.com/the-vandermonde-determinant-a-novel-proof-851d107bd728

Kariya, T., & Kurata, H. (2004). *Generalized Least Squares.* Hoboken: Wiley.

Karl Lundengård. (2017). Generalized Vandermonde Matrices and Determinants in Electromagnetic compatibility. *Mälardalen University Press Licentiate Theses* .

least squares fitting of a polynomial. (n.d.). Retrieved May 2022, from neutrium.net: http://neutrium.net/mathematics/least squares fitting of a polynomial

Least_squares#History. (2021, December). Retrieved January 2022, from wikipedia: https://en.wikipedia.org/wiki/Least_squares#History

polynomial regression. (n.d.). Retrieved May 2022, from wikipedia: http://en.wikipedia.org/wiki/Polynomial regression

Smith, K. (1918). On the Standard Deviations of Adjusted and Interpolated Values of an Observed Polynomial Function and its Constants and the Guidance They Give Towards a Proper Choice of the Distribution of the Observations. *Biometrika* , 1-85.

Stigler, S. M. (1974). paper on the design and analysis of polynomial regression experiments. *Historia MAthematica* , 431-439.

Value of the Vandermonde determinant/ Formulation 1. (2021, Septemer 11th). Retrieved May 19th, 2022, from proofwiki.org: https://proofwiki.org/wiki/Value_of_Vandermonde_Determinant/Formulation_1#:~:text=Let%20any%20xr%20be,x%20in%20f(x).

W, E. (2022, april 7). *least squares fitting--polynomial*. Retrieved april 12, 2022, from wolfcam.com: https://mathword.wolfcam.com

Wolberg, J. (2005). *Data Analysis Using the Method of Least Squares: Extracting the Most Information from Experiments.* Berlin: Springer.

Yin-Wen Chang; Cho-Jui Hsieh. (2010). Training and testing lowdegree polynomial data mappings via linear SVM. *Journal of Machine Learning Research* , 1471-1490.