Resolutivity and invariance for the Perron method for degenerate equations of divergence type

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Abstract

We consider Perron solutions to the Dirichlet problem for the quasilinear elliptic equation $\operatorname{div}\mathcal{A}(x,\nabla u)=0$ in a bounded open set $\Omega\subset\mathbf{R}^n$. The vector-valued function $\mathcal{A}$ satisfies the standard ellipticity assumptions with a parameter $1<p<\infty$ and a p-admissible weight w. We show that arbitrary perturbations on sets of (p,w)-capacity zero of continuous (and certain quasicontinuous) boundary data f are resolutive and that the Perron solutions for f and such perturbations coincide. As a consequence, we prove that the Perron solution with continuous boundary data is the unique bounded solution that takes the required boundary data outside a set of (p,w)-capacity zero.

Key words and phrases: capacity, degenerate quasilinear elliptic equation of divergence type, Dirichlet problem, Perron solution, quasicontinuous function, resolutive.

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1. Introduction

We consider the Dirichlet problem for quasilinear elliptic equations of the form

$$\operatorname{div}\mathcal{A}(x,\nabla u)=0 \tag{1.1}$$

in a bounded nonempty open subset Ω of the n-dimensional Euclidean space $\mathbf{R}^n$. The mapping $\mathcal{A}:\Omega\times\mathbf{R}^n\to\mathbf{R}^n$ satisfies the standard ellipticity assumptions with a parameter $1<p<\infty$ and a p-admissible weight as in Heinonen–Kilpeläinen–Martio [7, Chapter 3].

The Dirichlet problem amounts to finding a solution of the partial differential equation in Ω with prescribed boundary data on the boundary of Ω. One of the most useful approaches to solving the Dirichlet problem in Ω with arbitrary boundary